

Spatial evolution of short laser pulses under coherent population trapping

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Spatial and temporal evolution is studied of two powerful short laser pulses having different wavelengths and interacting with a dense three-level Lambda-type optical medium under coherent population trapping. A general case of unequal oscillator strengths of the transitions is considered. Durations of the probe pulse and the coupling pulse $T_{1,2}$ ($T_2 > T_1$) are assumed to be shorter than any of the relevant atomic relaxation times. We propose analytical and numerical solutions of a self-consistent set of coupled Schrödinger equations and reduced wave equations in the adiabatic limit with the account of the first non-adiabatic correction. The adiabaticity criterion is also discussed with the account of the pulse propagation. The dynamics of propagation is found to be strongly dependent on the ratio of the transition oscillator strengths. It is shown that envelopes of the pulses slightly change throughout the medium length at the initial stage of propagation. This distance can be large compared to the one-photon resonant absorption length. Eventually, the probe pulse is completely reemitted into the coupling pulse during propagation. The effect of localization of the atomic coherence has been observed similar to the one predicted by Fleischhauer and Lukin (PRL, **84**, 5094 (2000)).

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I. INTRODUCTION

The electromagnetically induced transparency (EIT) can be used to make optically thick media transparent to resonant laser radiation [1]. The EIT is the result of various quantum interference effects such as nonlinear interference [2], coherent population trapping (CPT) [3,4], adiabatic population transfer (APT) [5]. Optical characteristics of the matter undergo drastic changes under those effects to such an extent that they can now be manipulated. A lot of interesting applications based on that have been proposed and experimentally realized (see, e.g. [3–10]).

Interesting and unusual phenomena caused by the above indicated effects can be observed when laser pulses propagate in a resonant three-level medium. Propagation of pulses under EIT conditions was studied for example in [11–18]. As a rule, the situations are considered when both pulses have identical forms and their duration is longer than the relaxation time of the intermediate resonant state (matched pulses [11]; dressed field pulses [12,13]) or when the duration of the coupling radiation considerably exceeds that of the probe radiation (adiabatons [15,16,19]). A theoretical study of certain features of spatial evolution under APT conditions is presented in [16,17,20]. Propagation of soliton-like pulses in a three-level system is studied in [21]. A three-level system with equal oscillator strengths is under consideration in all the above mentioned studies, whereas in actual fact the transition oscillator strengths most often are different.

In this paper, the spatial and temporal evolution is studied of two overlapping short laser pulses propagating in a resonant optically thick medium that consists of three-level Λ -atoms. Pulses of such configuration are widely used to enhance the efficiency of nonlinear generation processes [22,23]. The two pulses are assumed to have identical shapes but different durations ($T_2 > T_1$) as shown in Fig.1. The pulse durations are much shorter than any of the times of relaxation in the medium (short pulses). It is also assumed that the pulse envelopes satisfy the adiabaticity criterion [13,24,25]:

$$(G_2\dot{G}_1 - G_1\dot{G}_2)/G^3 \ll 1, \tag{1}$$

where $G_{1,2}$ are the Rabi frequencies of the respective fields, $G = \sqrt{G_1^2 + G_2^2}$; the dot refers to time derivatives. Condition (1) is easy to satisfy by making one of the pulse amplitudes or both of them large even for short pulses. This will induce a strong coherence at the Raman transition resulting in the effect of CPT. The latter considerably decreases the absorption of the propagating resonant pulses. The dynamics of propagation of such pulses is studied here without restriction of the relationship between oscillator strengths of transitions.

Our theoretical model involves a set of coupled Schrödinger equations and a set of reduced wave equations, allowing thus a simultaneous description of temporal and spatial evolution of the atomic system and the radiation. The equations are analyzed in approximation (1) with the account of the first non-adiabatic correction. It will be shown that the dynamics of propagation strongly depends on the oscillator strengths ratio. Also the spatial and temporal behavior is analyzed of the atomic Raman coherence. A possibility of localizing the atomic coherence spatially has been established. The results obtained are compared to the results reported in [26].

The paper is organized as follows. In Section II, we describe the model and present the basic equations. Section III contains self-consistent solutions of those equations in the adiabatic limit and describes the temporal behaviour of the level populations and the atomic Raman coherence in the optically thin medium. The spatial evolution of pulses in the optically thick medium is described in Section IV for various oscillator strength ratios. In Section IV, we also discuss the adiabaticity criterion and demonstrate the effect of spatial localization of the atomic coherence. Finally we summarize the results obtained.

II. BASIC EQUATIONS

The three-level system under consideration is shown in Fig.1 together with the temporal configuration of the pulses as they enter the medium. The pulses travel along the same direction, z . States $|0\rangle$, $|1\rangle$ and $|2\rangle$ are connected by laser pulses $E_1 = 1/2 E_1(t) \exp[-i(\omega_1 t - k_1 z)] + c.c.$ and $E_2 = 1/2 E_2(t) \exp[-i(\omega_2 t - k_2 z)] + c.c.$, respectively. In our further consid-

eration we shall refer to the first pulse E_1 as the probe and to the second pulse E_2 as the coupling pulse. The Rabi frequency of probe pulse is comparable with that of the coupling pulse. The pulses are sent simultaneously into an atom. The pulse durations $T_{1,2}$ ($T_2 > T_1$) are assumed to be much less than any of the relaxation times of atoms. The transition $|0\rangle - |2\rangle$ is electric dipole forbidden. The intermediate state $|1\rangle$ is in a one-photon resonance with each field interacting only with the respective transition.

The following standard set of equations describes the spatial and temporal dynamics of the probability amplitudes of atomic states $b_{0,1,2}$ and slowly varying Rabi frequencies $G_1 = d_{10}E_1(t)/2\hbar$, $G_2 = d_{21}E_2(t)/2\hbar$ in the local-time coordinate system $\tau = t - z/c$:

$$\begin{aligned}\frac{\partial b_0}{\partial \tau} &= iG_1^* b_1 \exp(-ik_1 z), \\ \frac{\partial b_2}{\partial \tau} &= iG_2^* b_1 \exp(-ik_2 z), \\ \frac{\partial b_1}{\partial \tau} &= iG_1 b_0 \exp(ik_1 z) + iG_2 b_2 \exp(ik_2 z).\end{aligned}\tag{2}$$

$$\begin{aligned}\frac{\partial G_1}{\partial z} &= iK_1 b_1 b_0^* \exp(ik_1 z), \\ \frac{\partial G_2}{\partial z} &= iK_2 b_1 b_2^* \exp(ik_2 z).\end{aligned}\tag{3}$$

Here we assumed zero one-photon detunings. $K_{1,2} = \pi\omega_1 |d_{10,12}|^2 N/\hbar c$ are the propagation coefficients, N is the atomic concentration, $d_{10,12}$ are the dipole transition matrix elements, $\omega_{1,2}$, $k_{1,2}$ are the frequencies and the wave numbers of the interacting waves in vacuum, c is the light velocity in vacuum. All atoms are assumed to be initially in the ground state $|0\rangle$: $b_0(-\infty, z) = 1$, $b_{1,2}(-\infty, z) = 0$. We use Gaussian pulses at the medium entrance $z = 0$ for the purpose of numerical simulation: $G_1(\tau) = G_1^0 \exp(-\tau^2 \ln 2/T_1^2)$, $G_2(\tau) = G_2^0 \exp[-\tau^2 \ln 2/T_2^2]$.

In terms of $a_0 = b_0 \exp(ik_1 z)$, $a_2 = b_2 \exp(ik_2 z)$, $a_1 = ib_1$, equations (2) and (3) can be written as:

$$\begin{aligned}\frac{\partial a_0}{\partial \tau} &= G_1^* a_1, \\ \frac{\partial a_2}{\partial \tau} &= G_2^* a_1,\end{aligned}$$

$$\frac{\partial a_1}{\partial \tau} = -G_1 a_0 - G_2 a_2. \quad (4)$$

$$\begin{aligned} \frac{\partial G_1}{\partial z} &= -K_1 a_1 a_0^*, \\ \frac{\partial G_2}{\partial z} &= -K_2 a_1 a_2^*. \end{aligned} \quad (5)$$

The coupled equations (4) and (5) give a complete semiclassical description of the resonant different-wavelengths propagation problem we are dealing with.

III. TEMPORAL DYNAMICS OF LEVEL POPULATIONS AND RAMAN COHERENCE IN THE ADIABATIC APPROXIMATION (OPTICALLY THIN MEDIUM)

In this section we study the temporal dynamics of populations and the atomic coherence in the given time-dependent field, assuming that the medium is optically thin. Based on that, $G_{1,2}$ will not depend on the coordinate z . One can show that condition (1) for Gaussian pulses reduces to $G_2^0 T_1 \gg 1$ when $T_2/T_1 > \sqrt{2}$. With the first non-adiabatic correction, the solution of Eq. (4) takes on the form:

$$\begin{aligned} a_0 &\simeq \frac{G_2(\tau)}{G(\tau)}, \quad a_2 \simeq -\frac{G_1(\tau)}{G(\tau)}, \\ a_1 &\simeq \frac{1}{G_1} \frac{\partial(G_2/G)}{\partial \tau} \simeq -\frac{1}{G_2} \frac{\partial(G_1/G)}{\partial \tau}, \end{aligned} \quad (6)$$

where $G(\tau) = \sqrt{G_1^2(\tau) + G_2^2(\tau)}$.

The solutions for probability amplitudes are convenient to be represented as:

$$a_0 = \cos \theta(\tau), \quad a_2 = -\sin \theta(\tau), \quad (7)$$

where the mixing angle $\theta(\tau)$ is defined as $\tan \theta(\tau) = G_1(\tau)/G_2(\tau)$ (we shall discuss its meaning later on).

The expression for a_1 can be reduced to:

$$a_1 = (G_2 \dot{G}_1 - G_1 \dot{G}_2)/G^3 = \dot{\theta}/G. \quad (8)$$

In adiabatic limit (1) $|a_1| = |\dot{\theta}/G| \ll 1$ ($\dot{\theta} = \partial\theta/\partial\tau$), i.e. the population of the intermediate state $|1\rangle$ is close to zero all the time during the interaction with pulses. This also implies that the resonant absorption of the light pulses is weak (electromagnetically induced transparency) and the population is mainly distributed between the initial $|0\rangle$ and the final $|2\rangle$ states:

$$|a_0|^2 + |a_2|^2 \simeq 1. \quad (9)$$

Equality (9) reflects the fact that atoms are trapped in the CPT state: $a_{cpt} = (G_2/G)a_0 - (G_1/G)a_2 = a_0 \cos \theta - a_2 \sin \theta = 1$. This effect is responsible for the decrease in the resonant absorption of propagating pulses. Also the Raman coherence $\rho_{20} = a_0 a_2^*$ occurs:

$$\rho_{20} = -\frac{1}{2} \sin(2\theta) \quad \text{or} \quad \rho_{20} = -\frac{G_1 G_2}{G_1^2 + G_2^2}. \quad (10)$$

Obviously, the maximum coherence (in absolute value) $|\rho_{20}| = 1/2$ is reached when $\theta = \pi/4$ ($G_1^0 = G_2^0$). Figure 2 shows the temporal behavior of level populations $|a_{0,2}(\tau)|^2$, the atomic Raman coherence $|\rho_{20}(\tau)|$ and the mixing angle $\theta(\tau)$ for Gaussian pulses in the optically thin medium.

The above results can be interpreted in terms of the three-dimensional vector model where vector variables $\vec{a} = (a_0, a_2, a_1)$ and $\vec{G} = (G_2, G_1, 0)$ (the torque vector) are introduced. Using these variables, we can rewrite (4) as

$$\dot{\vec{a}} = \vec{G} \times \vec{a}, \quad (11)$$

where the sign ' \times ' means the vector product. The solution of equation (11) is vector $\vec{a} = (G_2/G, -G_1/G, \dot{\theta}/G)$. Components of the vector \vec{a} coincide with adiabatic solution (6). Figure 3 illustrates the dynamics of vectors \vec{G} and \vec{a} in the three-dimensional vector model. The torque vector \vec{G} moves in the \vec{e}_1 - \vec{e}_2 plane. And vector \vec{a} having a small angle with respect to vector \vec{G} ($|\dot{\theta}/G| \ll 1$) follows it. Thus one can see an absolute analogy with the adiabatic following in the case of a light pulse interacting with a two-level atom [27]. Such a simple picture can be observed only in optically thin media. In optically dense media, $G_{1,2}$ and hence θ become dependent on the z coordinate.

IV. SPATIAL EVOLUTION OF INTERACTING PULSES IN OPTICALLY DENSE MEDIA

A. General case: unequal oscillator strengths ($K_1 \neq K_2$)

The condition $K_1 \neq K_2$ means that the probability of the $|0\rangle - |1\rangle$ transition is not equal to that of the $|2\rangle - |1\rangle$ transition. We note that in the ideal adiabatic limit $a_1 = 0$, and the pulses would not change their shape as they propagate in a medium that is optically thick for each of the pulses (see (5)). However this is not the case. The non-adiabatic correction has to be introduced for real situations, which results in the induced dipole moments at the transitions $|1\rangle - |0\rangle$ and $|1\rangle - |2\rangle$, and in the change of both pulses traveling in the medium. In order to attribute this effect to propagation of the interacting pulses in an optically dense medium, it is necessary to solve Eqs. (4) and (5) in a self-consistent way.

Use (7) and (8) to rewrite field equations (5) in the following form

$$\frac{\partial G_1}{\partial z} = -K_1 \frac{\dot{\theta}}{G} \cos \theta, \quad \frac{\partial G_2}{\partial z} = K_2 \frac{\dot{\theta}}{G} \sin \theta. \quad (12)$$

From (12), one can show that

$$\begin{aligned} K_2 G_1^2(\tau, z) + K_1 G_2^2(\tau, z) \\ = K_2 G_1^2(\tau, z=0) + K_1 G_2^2(\tau, z=0), \end{aligned} \quad (13)$$

i.e. $K_2 G_1^2(\tau, z) + K_1 G_2^2(\tau, z)$ does not depend on the z coordinate.

Equation (13) describes the Manley-Row relation, i.e. the law of conservation of the total energy density during propagation under CPT conditions.

Using the definitions of θ and G , we obtain the following expressions for $G_{1,2}(\tau, z)$

$$\begin{aligned} G_1(\tau, z) &= G(\tau, z) \sin \theta(\tau, z), \\ G_2(\tau, z) &= G(\tau, z) \cos \theta(\tau, z). \end{aligned} \quad (14)$$

Substitution (14) into the Manley-Row relation gives the following expression for $G(\tau, z)$:

$$G^2(\tau, z) = G_0^2(\tau) \frac{K_2 \sin^2(\theta_0(\tau)) + K_1 \cos^2(\theta_0(\tau))}{K_2 \sin^2(\theta(\tau, z)) + K_1 \cos^2(\theta(\tau, z))}, \quad (15)$$

where $G_0^2(\tau) = G_1^2(\tau, 0) + G_2^2(\tau, 0)$ and $\theta_0(\tau) = \theta_0(\tau, 0)$ are the functions at the medium entrance $z = 0$.

So the dynamics of level populations as well as the atomic Raman coherence and the evolution of the pulse shape are completely determined by the function $\theta(\tau, z)$ that depends on both the time τ and the coordinate z . Differentiating $\tan \theta = G_1/G_2$ on the z and using (12) we obtain the following equation for $\theta(\tau, z)$:

$$\frac{\partial \theta}{\partial \tau} + \frac{G^2(\tau, z)}{K(\theta)} \frac{\partial \theta}{\partial z} = 0, \quad (16)$$

where $K(\theta(\tau, z)) = K_1 \cos^2(\theta(\tau, z)) + K_2 \sin^2(\theta(\tau, z))$.

Eq.(16) is similar to the equations describing nonlinear waves with the sharpening of the wave front during propagation [28]. The parameter $u = G^2/K$ can be treated as the "nonlinear" velocity. The nonlinear velocity can be described as $u(\tau, z) = A(\tau)/K^2(\theta(\tau, z))$ where the first factor $A(\tau) = G_0^2(\tau)[K_2 \sin^2(\theta_0(\tau)) + K_1 \cos^2(\theta_0(\tau))]$ is independent of the z coordinate, and the second factor $K(\theta(\tau, z))$ is maintained along a characteristic of Eq.(16) $\theta(\tau, z) = \text{const}$. This allows to write down the characteristic curve equation in an evident form:

$$z = \frac{1}{K^2(\theta_0)} \int_{\tau_0}^{\tau} A(\tau') d\tau' \quad (17)$$

Here τ_0 is the time at which the characteristic curve goes out of the medium boundary.

The solution for $\theta(\tau, z)$ has the form:

$$\theta(\tau, z) = \theta_0(\tau_0, 0) \quad (18)$$

Here τ_0 is to be determined from Eq.(17).

B. The case of equal oscillator strengths ($K_1 = K_2$)

In the case of equal oscillator strengths $K_1 = K_2 \equiv K$ the $G(z, \tau)$ function is not subject to changes during propagation ($G(\tau, z) = G_0(\tau)$ – the Manley-Row relation for this case).

Therefore Eq. (16) substantially simplifies:

$$\frac{\partial \theta}{\partial \tau} + \frac{G_0^2(\tau)}{K} \frac{\partial \theta}{\partial z} = 0. \quad (19)$$

The solution of Eq. (19) can be written in the following form:

$$\theta(\tau, z) = \theta_0(Z^{-1}(Z(\tau) - z), 0), \quad (20)$$

where $Z(\tau) = K^{-1} \int_{-\infty}^{\tau} d\tau' G^2(0, \tau')$, $Z^{-1}(\tau)$ is the inverse function of $Z(\tau)$.

It is not difficult to show that in this case $G = \sqrt{G_1^2(\tau, z) + G_2^2(\tau, z)}$ coincides with the definition of dressed field pulses [12,13]: $G_- = a_0 G_2 - a_2 G_1$. Thus the pulses in our case can be identified as dressed field pulses (only at $K_1 = K_2$). It would be interesting to note that other combination $G_+ = a_0 G_1 + a_2 G_2 \equiv 0$ (see also [12]). The concept of dressed field pulses cannot be applied to the case of $K_1 \neq K_2$, but in both cases the Manley-Row relation remains valid.

C. The adiabaticity criterion

The above results were obtained on the assumption that the adiabaticity criterion (1) (or $|\dot{\theta}/G| \ll 1$) remains valid during propagation of pulses. However it is not necessary the case. Therefore we investigate the adiabaticity condition with the account of propagation. Differentiating (18) on τ , we can write the following expression for the adiabaticity criterion:

$$\begin{aligned} \frac{\dot{\theta}(\tau, z)}{G(\tau, z)} &= \frac{\partial \theta_0}{\partial \tau_0} \frac{G(\tau, z)}{G_0^2(\tau_0)} \frac{K(\theta(\tau, z))}{K(\theta_0(\tau_0))} \\ &\times [1 + \sin(2\theta_0(\tau_0)) \frac{2(K_2 - K_1)z}{G_0^2(\tau_0)} \frac{\partial \theta_0}{\partial \tau_0}]^{-1} \ll 1 \end{aligned} \quad (21)$$

As follows from (21), the adiabaticity condition is destroyed ($\dot{\theta}/G \rightarrow \infty$) when the factor in the square brackets tends to zero. Since $\sin(2\theta_0(\tau_0)) > 0$ in the entire range of change of θ ($0 \leq \theta \leq \pi/4$), relation (21) is not fulfilled under the following conditions

$$\begin{aligned} \frac{2(K_1 - K_2)z}{G_0^2(\tau_0)} \frac{\partial \theta_0}{\partial \tau_0} \sin(2\theta_0(\tau_0)) &= 1, \\ (K_1 - K_2) \frac{\partial \theta_0}{\partial \tau_0} &> 0, \end{aligned} \quad (22)$$

Evidently, condition (22) is not fulfilled at $K_1 = K_2$, and the adiabaticity criterion holds throughout the propagation process in this case. It should be mentioned that numerical analysis of Eq. (22) reveals that, generally speaking, there is a range of change of the $q = K_1/K_2$ parameter when the first condition in (22) is not fulfilled either: $q_{min} < q < q_{max}$, $q_{min} < 1$, $q_{max} > 1$. The values q_{max} and q_{min} depend on the ratio $a = T_2/T_1$ and the shape of pulses. For example, the q parameter for Gaussian pulses with $T_2/T_1 = 3$ may vary within the limit $0.40 < q < 1.35$, as shown by the numerical simulation.

In this case for the thick medium the adiabaticity criterion has the form:

$$\frac{\partial \theta_0}{\partial \tau_0} \frac{G(\tau, z)}{G_0^2(\tau_0)} \ll 1. \quad (23)$$

It can be readily shown that condition (23) for Gaussian pulses is fulfilled when $T_2/T_1 > \sqrt{3}$ and $G_0^2 T_1 \gg 1$ (compare with the case of the thin medium).

For Gaussian pulses ($T_2 > T_1$) and the initial conditions $a_0(-\infty) = 1$, $a_2(-\infty) = 0$, we have $\dot{\theta}_0 > 0$ for $-\infty < \tau_0 < 0$. With $K_1 > K_2$ ($q > q_{max}$) the adiabaticity criterion begins to break down at the leading edge: the mixing angle front becomes steeper. Condition (21) gets destroyed at the trailing edge (where $\dot{\theta}_0 < 0$) when $K_1 < K_2$ ($q < q_{min}$). Let's introduce a critical length z_c at which the adiabaticity condition (21) cannot be satisfied. Using (22), one can obtain the following simple estimation for z_c for the case $K_1 \neq K_2$:

$$z_c \simeq \frac{G_0^2 T_1}{2|K_1 - K_2|}. \quad (24)$$

Let us now consider the experimental parameters: $N = 10^{15} \text{cm}^{-3}$; $1/\lambda_{1,2} = 10000, 20000 (\text{cm}^{-1})$; $1/\gamma_{1,2} = 10, 50 \text{ns}$ are the relaxation times of levels $|1\rangle$ and $|2\rangle$, respectively; $T_1 = 0.1 \text{ns}$; $G_{1,2}^0 T_1 = 20$; $a = T_2/T_1 = 3$; $f g_{10} = 0.1$ is the oscillator strength of the probe transition. From (24) we obtain $z_c = 10^5 z_0 \approx 2 \text{cm}$, where z_0 is the linear absorption length. This estimation agrees with the results presented in Figs. 5-7.

Using the characteristic equation (17) it is easy to find the area where the adiabaticity criterion breaks down. Characteristic curves for various values of $q = K_1/K_2$ are shown in Fig 4. The thickening of the characteristic curves means the sharpening of the mixing

angle front at a certain medium depth. At the point of intersection (see Fig. 4 b,c), $\dot{\theta} \rightarrow \infty$, condition (21) collapses. However there are no such points in the case when $q_{min} < q < q_{max}$ (see Fig.4a), i.e. here the adiabaticity criterion (21) is maintained during propagation of pulses.

The above discussion is illustrated by Fig.5 where the temporal behavior of the mixing angle θ is presented for $q = K_1/K_2 = 1, 0.25, 4$ and for different normalized propagation lengths. On the Fig.5a-b the analytical results are presented (formulae (18) and (20)). Figure 5c shows the numerical solution for θ at $K_1 > K_2$.

One can see that the evolution of the θ parameter at $K_1 > K_2$ is different from that at $K_1 < K_2$. At $K_1 > K_2$ the adiabaticity condition fails for all values of z , beginning from the critical lengths z_c defined by Eq. (24). Here, the analytical theory does not apply at the very late stage of nonlinear wave propagation. In the case of $K_1 < K_2$, nonadiabaticity develops at the trailing edge (the front becomes steeper at a certain propagation length), but it does not go deeper into the medium. A good agreement between the analytical and numerical solutions for θ at $K_1 \leq K_2$ is observed over the entire propagation length. This leads us to conclude that the interaction adiabaticity is fairly sensitive to the ratio of the oscillator strengths of the transitions interacting with the pulses.

D. Discussion of results

The solutions obtained have been used to analyze the temporal dynamics and spatial behavior of the propagating EIT pulses and the atomic coherence for various oscillator strength ratios. Figure 6 illustrates the temporal and spatial evolution of normalized Rabi frequencies of the both pulses $g_{1,2}(\tau) = G_{1,2}/\sqrt{G_1^{02} + G_2^{02}}$ as they propagate inside the optically thick medium. The temporal evolution of pulses can be seen to depend on the ratio between the transition oscillator strengths. In the case of $K_1 \leq K_2$ both pulses undergo reshaping as they propagate in the medium (Fig.6a and b). The probe pulse is gradually depleted and the coupling gets stronger. Note that the pulse shape at the initial stage of

propagation shows very little change along the length of the medium, which may exceed the linear absorption length. Complete reemitting of the probe pulse into the coupling one during propagation is possible. Using the Eq. (17) one can obtain the following expression for the maximal distance z_m on which the probe pulse propagates into the medium:

$$z_m = \frac{1}{K^2(\theta_0(-\infty))} \int_{-\infty}^{\infty} A(\tau) d\tau. \quad (25)$$

For the Gaussian pulses we have

$$z_m = \sqrt{\frac{\pi}{2\ln 2}} (G_2^0)^2 T_1 \frac{K_2(G_1^0/G_2^0)^2 + K_1 a}{K_1^2}. \quad (26)$$

Here $a = T_2/T_1$.

For the same parameters as in the previous sub-section, we have $z_m = 4.8 \cdot 10^5 z_0 \approx 9cm$ for $K_1/K_2 = 1$ and $z_m = 8.4 \cdot 10^5 z_0 \approx 16cm$ for $K_1/K_2 = 1/4$. These values agree with the results shown in Figs. 5-7.

An interesting feature of the spatial distribution of the probe pulse is illustrated in Fig. 6a,b: in some areas of the medium, the field in the tail of the pulse is different from zero. This is believed to be the result of the spatial compression of the probe caused by the slowing down of the group velocity of the probe pulse [26].

In the case of $K_1 > K_2$, the adiabaticity condition is maintained over the z_c range, which can also be much longer than the length of the linear absorption. The leading edge of the probe pulse undergoes gradual depletion and the pulse amplitudes display only small changes in that range. Outside that range, the pulse splits into several peaks. This occurs due to the nonadiabaticity of interaction (see Fig. 6c where numerical simulations are presented for $g_{1,2}(\tau)$. Here the first three curves correspond to the adiabatic interaction).

The pulse evolution, described above, is determined by spatial and temporal behaviour of the atomic Raman coherence ρ_{20} shown in Fig.7 (see also [18]). Figures 7a,b reveal an unusual spatial and temporal behavior of the atomic coherence, which we interpret as the slowing down, stopping and localization of the atomic coherence in the medium. The probe pulse is transferred into and stored in the collective atomic excitation under the

control of the coupling. One can say that a phase grating is created in the atomic medium. The phase grating is preserved throughout the entire period of relaxation of the atomic coherence. During that period of time, the information stored in the atomic excitations can be transferred back to the radiation using another coupling pulse of the same or of a different frequency.

We find these effects to be similar to the ones predicted and demonstrated in [9,10,26], but for some differences as indicated below. In our case, both the coupling pulse and the probe pulse are strong. Unlike [26], we used the boundary conditions for the pulse envelopes yielding the time distribution of pulses at the medium boundary, $z = 0$. We believe, this condition is more natural than the one used in [26] where the authors use the probe pulse distribution in the medium at fixed time as the initial condition. Also they do not take into consideration evolution of the coupling pulse. The effect takes place in the case $K_1 \leq K_2$ and is not observed in the case of $K_1 > K_2$.

V. CONCLUSION

The propagation of two short overlapping pulses with durations $T_2 > T_1$ in optically thick three-level media under CPT conditions has been studied for the general case of unequal transition oscillator strengths. An analytical solution has been obtained for the set of reduced wave equations under the adiabatic following condition. Also it has been shown how spatial evolution of pulses depends on the oscillator strength ratio.

The condition of adiabaticity provided at the medium entrance preserves for any value of propagation lengths if $K_1 \leq K_2$ ($q < q_{max}$) and breaks down at $K_1 > K_2$ ($q > q_{max}$). In the range $q < q_{max}$, the probe pulse is completely depleted and reemitted into the coupling pulse during propagation. This is not possible in the case of $q > q_{max}$. It has been established that to provide for the adiabaticity condition in an optically thin medium, the restriction $T_2/T_1 > \sqrt{2}$ has to be ensured, whereas in a thick medium $T_2/T_1 > \sqrt{3}$.

We also have studied the spatial behavior of the atomic coherence ρ_{20} , which plays a

significant role, for example, in nonlinear mixing processes. It has been found that a strong coherence can be maintained over a length equal to several hundreds of thousand of one-photon absorption lengths during propagation. The effect of localization of the atomic coherence is demonstrated.

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FIGURES

FIG. 1. (a) The three-level Lambda-type system coupled by two resonant pulses with Rabi frequencies G_1 and G_2 . (b) The shapes of probe G_1 and coupling G_2 pulses at the medium entrance.

FIG. 2. The time evolution of (a) level populations $|a_{0,2}(\tau)|^2$ and the atomic Raman coherence $|\rho_{20}(\tau)|$, (b) the mixing angle $\theta(\tau)$ in an optically thin medium for the Gaussian pulses. $G_1^0 T_1 = G_2^0 T_1 = 20$, (a) $T_2/T_1 = 3$; (b) $T_2/T_1 = \sqrt{2}$ (A), $\sqrt{3}$ (B), 3 (C), 10 (D).

FIG. 3. The vector model of adiabatic interaction of two short pulses with a three-level Lambda-type system.

FIG. 4. The characteristic curves for the Eq.(16): (a) $K_1/K_2 = 1$, (b) $K_1/K_2 = 0.25$, (c) $K_1/K_2 = 4$.

FIG. 5. The time evolution of the parameter θ for different relationships between K_1 and K_2 at different propagation lengths. (a) $K_1 = K_2$, (b) $K_1/K_2 = 0.25$, (c) $K_1/K_2 = 4$. $G_{1,2}^0 T_1 = 20$, $T_2/T_1 = 3$. Here and in all the other figures the time τ is measured in the units of the pulse duration T_1 , and the propagation length z of pulses in the medium is measured in the units of the length of linear absorption of the probe radiation determined in accordance with the Beer's law. In (c) the numerical solution for the case $K_1/K_2 = 4$ is presented.

FIG. 6. The time evolution of the normalized Rabi frequencies $g_{1,2} = G_{1,2}/\sqrt{G_1^{02} + G_2^{02}}$ of the probe and coupling pulses for different relationships between K_1 and K_2 at different propagation lengths. (a) $K_1 = K_2$, (b) $K_1/K_2 = 0.25$, (c) $K_1/K_2 = 4$. $g_{1,2}^0 = 20$, $T_2/T_1 = 3$.

FIG. 7. The time evolution of the atomic Raman coherence for different relationships between K_1 and K_2 at different propagation lengths. (a) $K_1 = K_2$, (b) $K_1/K_2 = 0.25$, (c) $K_1/K_2 = 4$. $G_{1,2}^0 T_1 = 20$, $T_2/T_1 = 3$.









